

CS/B.TECH(CSE)(N)/SEM-5/CS-503/2012-13

2012

DISCRETE MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

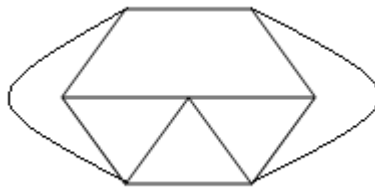
*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Question)

1. Choose the correct alternatives for the following: 10 x 1 = 10

i) What is the chromatic number of the following graph with 7 vertices?



- | | |
|------|-------|
| a) 6 | b) 5 |
| c) 4 | d) 3. |

ii) If there are n^r arrangements of r objects and n bins, then

- a) the objects and bins are all distinguishable
- b) the objects are distinguishable and bins are indistinguishable
- c) the objects are indistinguishable and bins are distinguishable
- d) the objects and bins are all indistinguishable

- iii) Consider the set A of all integers greater than 1. Let D be a relation defined on A by $(x, y) \in D$ iff x divides y . Then which of the following is true?
- a) D is both a lattice and a partial ordering
 - b) D is a lattice but not a partial ordering
 - c) D is neither a lattice nor a partial ordering
 - d) D is a partial ordering but not a lattice.
- iv) If 12 distinct points are placed on the circumference of a circle and all the chords connecting these points are drawn, at how many points do the chords intersect? Assume that no three chords intersect at the same point.
- a) $C(12, 2)$
 - b) $C(12, 4)$
 - c) 2^{12}
 - d) $12!/2$
- v) The set of natural numbers N with the relationship ' $|$ ' (divides) is a poset. How many minimal and maximal elements does it have?
- a) 1 minimal and 1 maximal
 - b) 1 minimal and 0 maximal
 - c) 1 minimal and more than 1 maximal
 - d) 0 minimal and 0 maximal

vi) What is the result of $(-3) \times_8 5 +_8 (-3) \times_8 (-5)$ in $[Z_8, +_8, \times_8]$, where Z_8 is the set of integers modulo 8, $+_8$ is the modulo 8 addition operation and $\times_8$ is the modulo 8 multiplication operation?

- | | |
|------|-------|
| a) 0 | b) 7 |
| c) 8 | d) 2. |

vii) How many ways are there to travel in xyz space from the origin $(0, 0, 0)$ to the point $(4, 3, 5)$ by taking unit steps in positive x, y, z direction only?

- | | |
|-------------------|---------------|
| a) $4!.3!.5!$ | b) 60 |
| c) $12!/(5!4!3!)$ | d) 3^{12} . |

viii) $A \wedge B$ is equivalent to which of the following?

- | | |
|--------------------------------|---------------------------------|
| a) $\neg A \rightarrow \neg B$ | b) $\neg A \rightarrow B$ |
| c) $\neg B \rightarrow A$ | d) $\neg(A \rightarrow \neg B)$ |

ix) A sequence $d = (d_1, d_2, d_3, \dots, d_n)$ is graphic if there is a simple undirected graph with degree sequence d . Which of the following degree sequences are graphic? Why?

$P: (2, 3, 3, 4, 4, 5)$

$Q: (2, 3, 4, 4, 5)$

- | | |
|--------------------|-----------------|
| a) Neither P nor Q | b) Both P and Q |
| c) P only | d) Q only |

- x) A complemented, distributive lattice is also called a Boolean Algebra. Consider a set $S = \{a, b, c\}$ and let $M = \wp(S)$ be the power set of S . Consider the inclusion (subset) relation ' $\subseteq$ '. Then $(M, \subseteq)$ is
- not a partial ordering
 - a partial ordering but not a lattice
 - a lattice but not a partial ordering
 - a Boolean algebra.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following.

$3 \times 5 = 15$

- C_9 is a cycle (i.e., a circular chain) with the nine vertices $a, b, c, d, e, f, g, h, i$. How many distinct maximal matchings of size four in C_9 contain the edge ab ?
- Consider K_6 , the complete graph on the six vertices a, b, c, d, e, f . The graph G_1 is obtained from K_6 by deleting the edge ab . The graph G_2 is obtained from G_1 by deleting the edge cd . What are the chromatic numbers of G_1 and G_2 ?
- A new flag is to be designed with 6 vertical stripes using 4 colours. In how many ways can this be done so that no 2 adjacent strips have the same colour?
- Give the sequence whose generating function is $g(z) = 5(z^5 - 1)/(z - 1)$.
- Consider the poset $S = \{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}$ under the relation ' $|$ ' (i.e. 'divides'). Find the following: Maximum element, Minimum element, Greatest element, Least element, $\text{lub}(2, 9)$, $\text{glb}(60, 72)$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following.

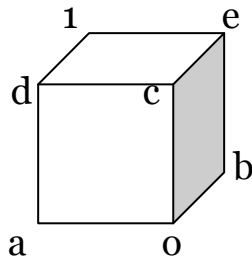
3 x 15 = 45

7. a) Show that s is a valid conclusion from the premises
 $p \rightarrow \sim q, q \vee r, \sim s \rightarrow p$.
- b) How many 10 bit binary strings are there none of which contains the pattern '110'?
- c) Use theory of congruence to prove that for $n \geq 1$,
 $17 \mid (2^{3n+1} + 3 \cdot 5^{2n+1})$.
8. a) Show that t is a valid conclusion from the premises $p \Rightarrow q$,
 $q \Rightarrow r, r \Rightarrow s$ and $p \vee t$.
- b) For any integer n , prove that the integer $8n+3$ and $5n+2$ are relatively prime. Hence find integers x, y such that
 $(8n + 3x) + (5n + 2y) = \gcd(8n+3, 5n+2)$.
- c) Define CRS (mod m) (complete residue system modulo m). Find all CRS (modulo 5).
9. a) Solve the recurrence relation:
 $a_{n+2} - 4a_{n+1} + 4a_{n-2} = (r + 1)2^r$
- b) Show that every bipartite graph is 2-chromatic.
- c) A positive integer n is expressed in the form $10b + b$.
Prove that n is divisible by 17 if $a - 5b$ is divisible by 17.

5 + 5 + 5

3 + 5 + 7

10. a) Show that the poset given in the following Hasse diagram is a lattice. Is it distributive and complemented? Justify your answer.



- b) Show that in a complemented distributive lattice $\langle L, \wedge, \vee \rangle$

i) $(a \wedge b)' = a' \vee b'$

ii) $(a \vee b)' = a' \wedge b'$

- c) Solve the following recurrence relation using generating function:

$$a_n = 4(a_{n-1} - a_{n-2}) + 2^n (n \geq 2); a_0 = 1, a_1 = 4.$$

$$6 + 4 + 5$$

11. a) Check the validity of the following arguments:

“If my program runs successfully then I will submit my project. I can appear the examination only if I submit my project. Either my program runs successfully or the computer crashes then I cannot appear in examination.”

- b) Define SDR of a family of finite sets. What is Hall's Marriage Condition? Consider the family of finite set $S = \{A_1, A_2, A_3, A_4\}$ where $A_1 = \{a, b, d, e\}$, $A_2 = \{b, c, d, e, f\}$, $A_3 = \{c, f\}$ and $A_4 = \{b, c, f\}$. Show whether S satisfies the marriage condition. If yes, find two void SDR of S .

- c) Write down the truth table for conditional and bi-conditional proposition.

$$5 + 5 + 5$$